Examiners' Report
Principal Examiner Feedback
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Pearson Edexcel International GCE
In Further Pure 3 (WFM03)
Paper : 01 Further Pure F3

General

Most students found questions they could attempt and there were many excellent solutions. The main challenges were in Q4, derivation of a reduction formula, Q6, calculation and use of an inverse matrix and Q8, calculation of the distance from a point to a line.
There was a reluctance amongst candidates to draw diagrams that would have clarified ideas in Q5, calculation of tangents to a hyperbola and Q8, calculation of distance from a point to a line. Writing down a formula for integration by parts (together with the separate components) or the formula $\cosh 2t = 2\cosh^2 t - 1$ before attempting to use them might have avoided the loss of accuracy marks.

Report on individual questions

Question 1

Most candidates made a good start to this question.
(a) Instructions to start with the definition $\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$ were overlooked by some who used $\sinh 3x = \sinh(2x + x) = \sinh 2x \cosh x + \cosh 2x \sinh x$. Most students tackling the problem correctly scored full marks. Sign errors in the expansion of the cubic $(e^x - e^{-x})^3$ or missing terms were seen. It is important that candidates who work on the left and right sides of the identity separately reach a conclusion "Shown", "√ ", "qed" or "lhs = rhs" are all acceptable.
(b) The result of part (a) was generally used to reach the cubic equation $4\sinh^3 x - 16\sinh x = 0$. Most students misread the question and only attempted the x values of the intersection of the two curves. The solution of $\sinh x = 0$ was frequently lost but solution of $\sinh^2 x = 4$ generally reached two correct values. $\sinh^2 x = 16$ was also seen. The easiest way of calculating y was 19(+/-sinh2) though the use of the exponential form for sinh $3x$ or $4\sinh^3 x + 3\sinh x$ was seen; hard work, though a few succeeded. Some candidates rejected $\sinh x = -2$. Values $\ln 2 + \sqrt{5}$ (missing brackets) and $\ln(\pm 2+\sqrt{3})$ were seen.
Students who used exponential functions rather than the result of part (a) were generally unsuccessful. Solution of $\sinh x = 2$ would have been quicker using the formula in the formula book rather than working through the exponentials.

Question 2

There were a high number of fully correct solutions. Candidates who completed the square of the quadratic independently of the integral generally made fewer errors.
(i) Occasional errors were made with the factor of three and a final answer was occasionally short of 1/6 in front of the function or ½ inside the bracket.
(ii) The negative x^2 caused a number of problems. Writing $27 - 6x - x^2 = -(x^2 + 6x - 27)$ would have been beneficial. $\int 1/\sqrt{(27 - 6x - x^2)}\, dx = - \int 1/\sqrt{(x^2 + 6x - 27)}\, dx$ was seen a number of times with the loss of all four marks.
Correct inverse functions were generally recognised. In (i) the ½ or 1/6 were occasionally absent inside the brackets and $3(x^2 + 4x + k)$, $k \neq 8$ was seen.

Question 3

Whilst a few students were unhappy about matrix algebra many completely correct solutions were seen.
In part (a) the main approach to achieving $k = 2$ was to expand det($\mathbf{M} - \lambda\mathbf{I}$)=0 , put λ=3 and solve a simple equation. A few chose to expand the quadratic in λ and k before introducing $\lambda = 3$. This was open to errors, especially with bracketing. An incorrect value for k made life difficult in the remaining parts of the question. A few approached the solution by writing $\mathbf{Mx} = 3\mathbf{x}$, expanding and solving the equations. $k = 2$ was generally found although this method produced numerical errors.
Many solutions for part (b) reached the eigenvalues 1 and 2. When using $k = 2$ the solutions that removed the factor $(3 - \lambda)$ found the algebra simplified. Most candidates were able to factorise their cubic correctly though for some previous errors meant $(3 – \lambda)$ was not a factor.
In part (c) good progress was made finding an eigenvector though some students wrote down correct values for x, y and z but then listed them in the wrong order in the vector (e.g. $x = y$, $z = 2y$ leading to $(2\ 2\ 1)^T$).The request to find a unit vector was occasionally overlooked.
A few solutions calculated the eigenvector linked to $\lambda= 1$ instead of $\lambda = 3$.

Question 4

There were many excellent solutions to this question. Some candidates made little progress in part (a) but achieved marks in (b).
The final accuracy mark was occasionally lost in (a) through poor bracketing or missing a function in the final line. A few solutions split the parts as x^{n-1} and $x\cos x$. The second integration by parts caused a few of these attempts to falter.
Part (b) was well done by most candidates. The general method was to express I_4 in terms of I_2 and then I_2 in terms of I_0. $I_0$was occasionally evaluated as 0, 1 or x. The final answer occasionally had one of the x^n terms with a wrong power or $\sin x$/$\cos x$ absent. Sometimes a value of I_0 was embedded in the work and it was difficult to establish its value when the various multiples were mixed up.
A few candidates decided to ignore the result of part (a) and used repeated integration by parts. They were generally successful.

Question 5

In part (a) most solutions correctly substituted $y = mx + c$ into the equation of the hyperbola to reach a correct, simplified quadratic equation. Use of the discriminant was well known and, apart from a few careless errors the algebraic working reached the required $25m^2 = c^2 + 4$.
A few candidates tried a parametric approach but often failed to make a link between m and c and their parametric expressions for them. Quite often it was assumed that (1, 2) was a point on the hyperbola and a straight line found there. A handful of solutions succeeded using parameters ($5\sec t$, $2\tan t$) or ($5\cosh t$, $2\sinh t$).
Part (b) caused problems with a number of candidates making no further progress with this question. Candidates who wrote down the equations $25m^2= c^2 + 4$ and $2 = m+ c$ generally

solved them correctly and reached two correct tangents. Use of the quadratic formula was quite common though simplifying the quadratics led to easy factorisation.
Some candidates who had scored well up to this point found part (c) challenging. Substitution of one of their tangents into the hyperbola equation led to tricky work with fractions in order to reach a quadratic with equal roots. A number of solutions followed the required method but made sign errors in the coordinate values.

Question 6

Part (a) showed that some candidates seemed unfamiliar with basic matrix work and failed to apply a recognised method to calculate $\mathbf{A}^{-1}$. A correct expression for the determinant was generally seen though simplification to $2a$ or $2a - 1$ rather than $2a - 2$ was seen. A few attempts at $\mathbf{A}^{-1}$ calculated all the relevant determinants but then failed to use appropriate signs. There were occasional slips in the entries which meant the solution became messy in part (b)
(b) Four methods were seen for transforming l_2 to l_1, three of which used the inverse matrix.
(i) The most popular using the parametric form of l_2 $(6 - \lambda, -4 + 4\lambda, 2 - \lambda)^T$ was generally done accurately but an incorrect value of the determinant lost three accuracy marks overall. The algebra was much simpler where the multiple of 1/6 was left outside the matrix until the end. The final mark was often lost when "line is" or "l =" was used rather than the vector form "r =". Occasionally the vector product form of the line was given which was fine.
(ii) Transforming a point on the line and its direction were quite common and generally presented few problems.
(iii) Transforming two points on the line was seen and was generally successful; the simpler the points chosen, the less likelihood of arithmetical slips.
(iv) A small number of students took a point (x, y, z) on l_1, transformed it with $\mathbf{A}$ and made the resulting point lie on line l_2. The algebra was complicated, and many gave up, though a few solutions did reach the required straight line.

Question 7

In part (a) many good solutions to verifying $(\mathrm{d}x/\mathrm{d}t)^2 + (\mathrm{d}y/\mathrm{d}t)^2 = 2\cosh^2 t$ were seen. $\mathrm{d}x/\mathrm{d}t$ and $\mathrm{d}y/\mathrm{d}t$ were correct in most cases and the expressions squared accurately. There were some careless slips in reaching $2\sinh^2 t + 2$. The final accuracy mark was often lost as an intermediate step $2\sinh^2 t + 2 = 2(\sinh^2 t + 1)$ or $2\sinh^2 t + 2 = 2(\cosh^2 t - 1) + 2$ was required since this was a "show that" question.
The derivation of the formula for surface area was well done in part (b) though the occasional bracketing error lost the accuracy mark.
Evaluation of the integral in part (c) caused some difficulties. Candidates who attempted the derivation of a four term expression for an indefinite integral generally avoided sign errors. Evaluation of $\cosh^2 t$ was generally done using the double angle formula though a failure to write down $\cosh 2t = 2\cosh^2 t - 1$ led to sign errors or a missing ½. Attempts using exponentials made progress though did cause difficulties later. Integrating $t\cosh t$ by parts was well done though errors arose due to a failure to show full working. Solutions which evaluated the definite integrals separately often lost accuracy marks when combining as they overlooked the fact that the second part was $-\int t\cosh t\,\mathrm{d}t$.

The last two marks for evaluation of the integral were also lost by the assumption that the lower limit value was zero.

Question 8

Most students made some progress in part (a) and the two methods, finding the direction of the line and a point on the line or solving the equations of the planes simultaneously were equally popular.
Evaluation of vector product $(1\ -5\ 3)^T \times (3\ -2\ 2)^T$ was generally accurate. Calculation of a point on the line starting with either x or y or $z = 0$ was also generally accurate though starting with different values saw some arithmetical slips. The final accuracy mark was occasionally lost due to the omission of "**r** =" from the equation.
The method where students aimed to find a formula for z in terms of y and z in terms of x (for example), proved more challenging and careless errors were quite common. Some solutions stopped with just one equation though many students managed to reach either a Cartesian equation or a vector equation of the line, albeit with numerical errors.
Part (b) was possibly the most challenging question on the paper. Many students did not explain their strategy clearly. Many solutions only scored the first mark for calculating **PQ**. A diagram would have been helpful.
(i) The "easiest" method used a vector product of $(1\ 2\ -2)^T$ and $(-4\ 7\ 13)^T$ to calculate $\sin\theta$ (where θ is the angle between PQ and the line) followed by multiplication by the length of *PQ*. Not a popular method though students using this approach made few errors.
(ii) The most common method was to use a scalar product to calculate $\cos\theta$. Many used the two position vectors of *P* and *Q* rather than **PQ** and the direction of the line. Evaluation of a correct scalar product was often achieved. Many thought that multiplication by 3 would give the answer rather than the length of the projection of **PQ** on the line. Calculation of $3\sqrt{(1-\sin^2\theta)}$ to reach an exact answer proved tricky.
A relatively small number of candidates reached one of many equivalent exact values such as $5\sqrt{74}/234$, $5\sqrt{481}/39$, $(5\sqrt{37})/3\sqrt{13})$, $\sqrt{(925/117)}$ etc
(iii) A few solutions attempted **NP** (where N is the foot of the perpendicular from P to the line), calculated NP^2 as a function of the parameter and then aimed to minimise either by calculus or completing the square. Only careless arithmetic prevented a successful solution.
(iv) **NP** was also used in the calculation $\mathbf{NP}.(-4\ 7\ 13)^T = 0$ to find the parameter and then distance.
A few solutions used the formula for the calculation of the distance from a point to a plane.